Mathematical Modeling of Subgenomic Hepatitis C Virus Replication in Huh-7 Cells

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Cell-based hepatitis C virus (HCV) replicon systems have provided a means for understanding HCV replication mechanisms and for testing new antiviral agents. We describe here a mathematical model of HCV replication that assumes that the translation of the HCV polyprotein occurs in the cytoplasm, that HCV RNA synthesis occurs in vesicular-membrane structures, and that the strategy of replication involves a double-stranded RNA intermediate. Our results shed light on the intracellular dynamics of subgenomic HCV RNA replication from transfection to steady state within Huh-7 cells. We predict the following: (i) about $6 \times 10^3$ ribosomes are involved in generating millions of HCV NS5B-polymerase molecules in a Huh-7 cell, (ii) the observed 10:1 asymmetry of plus- to minus-strand RNA levels can be explained by a higher-affinity (200-fold) interaction of HCV NS5B polymerase-containing replication complexes with HCV minus-strand RNA over plus-strand RNA in order to initiate synthesis, (iii) the latter higher affinity can also account for the observed ~6:1 plus-strand/minus-strand ratio in vesicular-membrane structures, and (iv) the introduction of higher numbers of HCV plus-strand RNA by transfection leads to faster attainment of steady-state but does not change the steady-state HCV RNA level. Fully permissive HCV replication systems have been developed, and the model presented here is a first step toward building a comprehensive model for complete HCV replication. Moreover, the model can serve as an important tool in understanding HCV replication mechanisms and should prove useful in designing and evaluating new antivirals against HCV.

About 200 million people, roughly 3% of the human population, are infected with hepatitis C virus (HCV) (61). Chronic HCV infection is the main cause of chronic liver disease and cirrhosis, leading to liver transplantation or death (3, 47). State-of-the-art therapy (peg-interferon and ribavirin) elicits long-term responses in only about 50% of treated patients (18, 37, 41), with no effective alternative treatment for nonresponders (56).

Progress toward developing model systems of HCV infection that could enhance efforts to identify inhibitors of HCV replication has been hampered by HCV’s limited replication in cell culture and the lack of small animal models (32). In 1999 Lohmann et al. (35) engineered a bicistronic subgenomic HCV replicon system in Huh-7 cells. Since then this system, improved substantially both in Huh-7 cells (34) and in other cell lines (64), has become the standard cell-based assay to study HCV replication mechanisms and to evaluate antiviral agents (51).

The first studies of positive-strand RNA virus replication were done with RNA bacteriophages, e.g., Qβ and MS2 (54). These studies showed that viral RNA amplification depended on an RNA-dependent RNA polymerase-containing RNA replicase that specifically interacts with the incoming viral RNA (plus strand) to synthesize its complementary (minus) strand. Once the minus-strand RNA is synthesized, the amplification of the viral RNA by the replicase begins. Based on these systems, Biebricher et al. (6) quantitatively monitored the kinetics of RNA amplification by Qβ replicase and developed a kinetic model for self-replication of Qβ RNA in vitro (5). The full life cycle of Qβ has been mathematically modeled (16) and provides an important starting point for developing intracellular HCV replication models.

HCV is an enveloped positive-strand RNA virus belonging to the genus Hepacivirus in the family Flaviviridae (32). After HCV enters a cell, the HCV genome is translated, by host ribosomes, into a large polyprotein, about 3,000 amino acids long, that is processed into structural and nonstructural (NS) proteins. A multiprotein viral replicase is assembled from the NS proteins (35) and begins the synthesis of a minus-strand RNA using the positive single-strand RNA (ssRNA) as a template. Once the minus-strand RNA is synthesized, it can remain as a free minus ssRNA or be attached to the positive-strand RNA to form a double-stranded RNA (dsRNA). The newly synthesized minus-strand RNA (either as ssRNA or as part of the dsRNA) then serves as a template for the synthesis of additional plus-strand RNAs. It has not yet been determined whether HCV RNA in cell culture (or in vivo) is replicated using as a template the minus-strand RNA in a dsRNA form, as established for the Kunjin virus (10), or as a ssRNA template, as shown for the Qβ phage (16). In the present study, we make the assumption that after the first minus strand is replicated, amplification of viral RNA occurs via a double-stranded template.

All positive-strand RNA viruses replicate their RNA on intracellular membranes, often in association with spherular invaginations of the target membrane (reviewed in reference 53). RNA replication by Kunjin virus, coronaviruses, brome mosaic virus, and poliovirus induces distinct membrane rear-

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arrangements of 50 to 350 nm, including invaginations, double-membrane vesicles, and layered membranes, that serve as compartments or minioranelles for RNA replication (53). Membrane association of the HCV NS5B polymerase was found to be essential for HCV RNA replication (15, 39), and HCV RNA replication complexes colocalize with vesicular-membrane structures (VMS) that have also been termed “the membranous web” (15, 22). Once formed, these membrane structures appear to be relatively stable, with only limited movement and exchange of viral NS proteins (60a).

The experimental characterization of HCV self-activation in the replicon system in Huh-7 cells reveals that (i) by 24 to 72 h posttransfection, plus-strand RNAs accumulate to ∼5,000 copies per cell (8, 30, 34, 35, 50); (ii) the plus-strand/minus-strand ratio is about 10:1 (35, 50), which is in agreement with the plus-strand/minus-strand ratio observed in infected hepatocytes in humans (9, 27); (iii) the in vitro replicase activity, prepared from Huh-7 cells harboring subgenomic replicons, is highly resistant to nuclease and protease treatment, with both plus- and minus-strand RNAs being fully nuclease resistant (50); and (iv) <5% of the NSSB polymerase molecules are protease resistant (38, 50), suggesting that the majority of replication occurs in sites, such as the VMS, that are protected from nuclease and protease activity, and where a minority of NSSB polymerase may reside.

Recently, we have developed mathematical models to gain insight into HCV RNA dynamics during primary infection (13), liver transplantation (12, 46), alpha interferon (IFN-α) monotherapy (40), and IFN-α and ribavirin combination therapy (14). However, these models were not designed to gain insight into HCV minus- and plus-strand kinetics within the cell. Based on quantitative data for subgenomic HCV replication in Huh-7 cells, we sought to gain a better understanding of its dynamics from transfection to steady state using a mathematical model.

MATERIALS AND METHODS
Model description. Our model of subgenomic HCV replication in Huh-7 cells is based on the HCV replication scheme shown in Fig. 1. We assume that translation of the HCV polyprotein occurs in the cytoplasm by host ribosomes, is based on the HCV replication scheme shown in Fig. 1. We assume that the enzyme NS5B and associated viral proteins needed for HCV RNA synthesis, HCV RNA molecules, translation complexes, HCV polyprotein molecules, and for simplicity, we assume that 10 ribosomes simultaneously translate the same RNA, and that the ratio of plus-strand to minus-strand RNA is 6:1 (50), (iv) the steady-state is established when translation is complete. Free plus-strand RNAs in the cytoplasm can be degraded by nucleases at rate μp, lost from the cytoplasm by transport into VMS at rate kRout, and gained by transport out of the VMS at rate kRin. We assume that free viral polyprotein molecules, P, are generated by translation within the cytoplasm at rate kp per translation complex and are cleaved into separate viral proteins, including the enzymes responsible for HCV RNA synthesis, Ecp, e.g., NS5B, at rate kE. Lastly, we assume these enzymes may be degraded before reaching the VMS at rate kdp, or transported into VMS at rate kRin.

\[
\frac{dR_+}{dt} = -k_R R_+ + k_p R_{10} + k_p R_{10} - k_{pR} R_{10} - \mu_p R_{10}
\]

\[
\frac{dR_{-}}{dt} = k_p R_{10} + k_p R_{10} - k_{pR} R_{10} - \mu_R R_{10}
\]

\[
\frac{dP}{dt} = k_T R_{10} - k_p P
\]

\[
\frac{dE}{dt} = k_p P - k_{Ecp} E + \mu_E E
\]

Equations 5 to 9 describe the kinetics of HCV RNA replication within the VMS, with R+, R−, and E representing the numbers of plus-strand RNA, dsRNA, and HCV polymerase complexes, respectively, within the VMS. Rcp and Rds represent the numbers of plus-strand RNA and dsRNA replicative intermediate complexes, respectively. The Rcp complex is composed of a plus strand that serves as a template for the newly synthesized minus-strand RNA, the replication machinery (E), and the nascent complementary minus-strand RNA. The Rds complex contains dsRNA in which the minus strand serves as a template for the newly synthesized plus-strand RNA, the replication machinery E, and the nascent complementary plus-strand RNA. We assume that once the synthesis of the nascent strand has ended, the replication complex, Rcp, immediately dissociates into dsRNA (consisting of plus- and minus-strand RNAs), Rds, and the replication machinery E. Finally, when the synthesis of the nascent plus strand has ended, the replication complex, Rcp, immediately dissociates into three components: the unwound plus strand, the replication machinery (E), and dsRNA. De novo formation of Rcp and Rds complexes in VMS occur with rate constants k1 and k2, respectively, and degrade with rate constants μp and μE, respectively. Synthesis of the nascent plus- and minus-strand RNAs occurs with rate constants kΔ and kΔE, respectively. Free plus-strand RNA and dsRNA in VMS (R+ and R−) degrade with rate constants μp and μR, respectively. In addition, the polymerase complex degrades or loses activity with rate constant μp, within VMS, which is possibly less than the cytoplasmic degradation rate μp. We assume that there are abundant nucleotides and amino acids in the cell so that cellular resources do not limit HCV RNA replication (55). We also assume that if any cellular components are needed to form the replication machinery (E), they, too, are abundant. This should be the case in the stage of subgenomic HCV replication being considered here, when Huh-7 cells are replicating and have not yet reached confluence.

Simulation procedure. We have computed the solutions to equations 1 to 9 numerically using the Rosenbrock algorithm for solving stiff differential equations, implemented in C++ (48). The model was solved 1,000 times with different (randomly chosen) parameter sets in each run. The results of each run were stored and analyzed for consistency with the biological criteria discussed below. At the time of transfection (t = 0), we assume that RΔ(0) ≥ 1 replication-competent plus-strand RNAs are introduced into a cell and that all other virus variables are zero (T = P = E = Rcp = Rds = Rcp = Rds = 0).

The parameters k1, k2, kΔ, kΔE, μp, and μE were estimated from the literature as described below. The remaining parameters were chosen (Table 1) so the model reaches a steady state with the following experimental characteristics: (i) plus strands are at a level of about 900 to 5,000 copies per cell (23, 35, 50), (ii) the total plus-strand/minus-strand ratio is about 10:1 (23, 35, 50), (iii) the plus-strand/minus-strand ratio in VMS is , (iv) the steady-state is established about 48 h posttransfection (34, 35, 50), (v) the number of NSSB molecules is between 8 × 10^7 to 2 × 10^8 per cell (50), and (vi) a small proportion (<5%) of HCV NS proteins expressed in cells harboring HCV replicons are actively en
engaged in HCV RNA synthesis (38, 50). Thus, we assume, in our model, that the NS5B polymerase molecules in the cytoplasm represent at least 95% of the total NS5B polymerase molecules in a cell with the rest (≤5%) being present in VMS.

(vii) Moreover, about half of total plus-strand RNAs in a replicon cell are nuclease resistant (50) and thus localize (in our model) to the VMS. The plus-strand RNAs that are present in the cytoplasm are assumed to be involved in viral polyprotein translation.

**Polysome size and HCV polyprotein elongation rate.** The elongation rate in eukaryotes has been estimated at three to eight amino acids per second per ribosome (33, 43). At three to eight amino acids per second, the subgenomic HCV polyprotein (~2,000 amino acids) is translated at a mean rate of 10 (range, 5.4 to 14.4) polyproteins per h per ribosome. Wang et al. (59) have shown that at least eight ribosomes were present on an efficient replicon HCV RNA during translation. As shown in the Appendix, one can use a complex model that includes the sequential attachment of multiple ribosomes to each HCV mRNA. However, we found that it is feasible to simplify the situation and use the model described here when the rate constant for the attachment of the first ribosome to free plus-strand RNA is much lower than the rate constant for subsequent ribosomes to attach. This may be reasonable since the rRNA helicases need first to unwind the secondary structure of the mRNA (25), which then might lead to

FIG. 1. Schematic model of subgenomic HCV replication in Huh-7 cells. HCV replication starts once plus-strand RNA, $R_{+}^{o}$, enters the cell during transfection. The plus strand interacts with ribosome complexes to form the translation complex ($T_c$) with rate constant $k_1$. Once $T_c$ is formed, translation begins and the viral polyprotein ($P$) is produced at rate $k_2$. After the polyprotein is produced, we assume the ribosome complex dissociates from $T_c$, leading to a free plus-strand RNA. The resulting polyprotein is cleaved with rate constant $k_c$ into separate viral proteins, including the NS5B polymerase ($E_{cyt}$) (containing RNA replicase) that is transported into the VMS at rate $k_{Ein}$. The plus-strand RNAs in the cytoplasm ($R_{P}^{o}$) are transported into VMS with rate constant $k_{Pin}$ and out of VMS with rate constant $k_{Pout}$. Within the VMS, the association of plus-strand RNA ($R_P$) and NS5B ($E$) result in the formation of the plus-strand replicative intermediate complex ($R_{IP}$) that occurs at rate $k_{3RPE}$. The complementary minus-strand RNA is then formed with rate constant $k_{4m}$ and the $R_{IP}$ complex is dissociated to dsRNA ($R_{ds}$) and NS5B polymerase ($E$). Finally, when dsRNA is present, the formation of the dsRNA replicative intermediate ($R_{Ids}$) occurs at rate $k_{5RdsE}$ and replicates nascent plus-strand RNA at rate $k_{ds}$ per complex. Once the full nascent plus RNA is replicated, the unwound plus RNA is released from the $R_{Ids}$ complex (along with $R_{ds}$), and $E$. Ribosomes are indicated as open black squares, plus-strand RNAs are indicated as black lines, minus-strand RNAs are indicated as dotted black lines, NS5B polymerase are indicated as open black ovals, plus-strand polymerase intermediate ($R_{IP}$) and dsRNA replicative-intermediate complexes are colored green. Kinetic rates $k_1$ to $k_5$, $k_{Pin}$, $k_{Pout}$, $k_c$, and $k_{Ein}$ are indicated as black arrows. Degradation rates of the NS5B polymerase (in cytoplasm [$\mu_{E}^{o}$] and in VMS [$\mu_{E}$]), plus and dsRNA replicative-intermediate complexes ($\mu_{IP}$ and $\mu_{Ids}$, respectively), plus-strand RNAs (in cytoplasm [$\mu_{P}^{o}$] and in VMS [$\mu_{P}$]), translation complex ($\mu_{Tc}$), and dsRNAs ($\mu_{ds}$) are indicated as blue arrows. Red arrows represent the dissociation of the replicative-intermediate complexes ($R_{IP}$ and $R_{Ids}$) and translation complexes ($T_c$) immediately after the full synthesis of the nascent RNA strands and viral polyprotein is finished. The vesicular-membrane structures observed in Huh-7 cells (15, 22), are represented here by just one VMS marked with a large dotted pink oval. The VMS size is not drawn to scale in relation to the Huh-7 cell.
a faster attachment of the subsequent ribosomes. We thus used the simplified model and fixed the polyosome size at 10 ribosomes per HCV mRNA, yielding a subgenomic HCV polyprotein translation rate $k_p = 10^2$ polyproteins per h per polysome.

**HCV plus- and minus-strand RNA synthesis rate.** HCV RNA has been estimated to be synthesized at approximately 150 nucleotides (nt) per min by HCV recombinant NS5B purified from Escherichia coli (42) and up to 180 nt/min in Huh-7 cells (36). Thus, the synthesis rate for subgenomic plus or minus HCV RNA (~6,300 nt [35]) is $k_s = 1.7$ RNA molecules per h per replicative intermediate complex. We assume that there is no difference in synthesis rate when the minus strand (as part of dsRNA) or plus strand serves as a template for replication (i.e., $k_s = k_{s\text{p}} = 1.7$).

**HCV plus- and minus-strand RNA degradation rates.** Plus- and minus-strand RNAs involved in replication complexes are fully resistant to nuclease treatment (50) and are assumed, in our model, to degrade at a slower rate than free RNAs involved in replication complexes. Some HCV RNA complexes (corresponding to 5,000 to 10,000 some-HCV RNA complexes) dissociate immediately.

**NSBS polymerase degradation rate in cytoplasm.** More than 95% of NSBS polymerase molecules in vitro were shown to be sensitive to protease treatment and not involved in replicase activity (38, 50). Thus, it is likely that the observed NSBS polymerase half-life (44, 45, 59) corresponds to the NS5B polymerase half-life, 12 h (44, 45, 59), which is significantly faster than the observed overall plus-strand RNA degradation rate ($k_{s\text{p}}$) is significantly faster than the observed overall plus-strand RNA degradation rate ($k_{s\text{p}}$).

### RESULTS

Based on current knowledge of related positive ssRNA viruses and bacteriophage replication mechanisms, we developed a model for subgenomic HCV replication in Huh-7 cells described in Materials and Methods. Quantitative studies of HCV replication both in cell culture and in vitro allowed us to minimize the uncertainty of model parameters and provide experimental data to compare with simulation results. We then estimated unknown host and viral components and kinetic constants involved in subgenomic HCV replication within Huh-7 cells. We used our model to explore the biological basis of the plus-strand to minus-strand asymmetry and the overproduced NS proteins found in replicon cells.

**Number of ribosomes available for HCV translation.** We found that the number of ribosomes available for HCV translation ($R_{\text{plus}}$) is the only parameter that significantly affects the number of NS5B polymerase molecules in the cytoplasm (Fig. 2A). To obtain approximately a million NS5B polymerase molecules (50), we needed to assume that 500 to 1,000 plus- and minus-strand RNA complexes (corresponding to 5,000 to 10,000 ribosomes) are involved in HCV translation at steady state.

**1:1 ratio of plus-strand RNA inside and outside the VMS.** Approximately half of the total plus-strand RNAs in a replicon cell is nuclear resistant (50) and thus assumed in our model to localize in the VMS. To obtain a total number of plus-strand RNAs in the cytoplasm ($R_{\text{plus}} + T_c$) equal to the total number of plus-strand RNAs in VMS ($R_{\text{plus}} + R_{\text{p}} + R_{\text{Ids}} + R_{\text{ds}}$), we

### TABLE 1. Parameter estimates of subgenomic HCV replication in Huh-7 cells

<table>
<thead>
<tr>
<th>Rate constant</th>
<th>Reaction definition</th>
<th>Kinetic rate constants (h⁻¹):</th>
</tr>
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<tbody>
<tr>
<td>$k_s$</td>
<td>Nascent NS polyprotein translation</td>
<td>1-100† (molecule⁻¹)</td>
</tr>
<tr>
<td>$k_{s\text{p}}$</td>
<td>Viral polyprotein cleavage</td>
<td>100*</td>
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<td>$k_{p\text{m}}$</td>
<td>$R^+_{s\text{m}}$ transport into VMS</td>
<td>0.2–1†</td>
</tr>
<tr>
<td>$k_{p\text{d}}$</td>
<td>$R^+_{s\text{d}}$ transport into cytoplasm</td>
<td>0.2†</td>
</tr>
<tr>
<td>$k_{\text{ds}}$</td>
<td>$E^{+\text{d}}$ transport into VMS</td>
<td>4.0 x 10⁻⁶ to 4.0 x 10⁻⁵†</td>
</tr>
<tr>
<td>$k_{\text{ps1}}$</td>
<td>$R^+_{\text{ps1}}$ formation</td>
<td>12 x 10⁻⁵ to 3.3 x 10⁻⁵‡</td>
</tr>
<tr>
<td>$k_{\text{ps2}}$</td>
<td>$R^+_{\text{ps2}}$ synthesis</td>
<td>0.001–0.02‡ (molecule⁻¹)</td>
</tr>
<tr>
<td>$k_s/k_{s\text{p}}$</td>
<td>$R^+_{\text{p}}$ degradation</td>
<td>1.7*</td>
</tr>
<tr>
<td>$k_s/k_{s\text{d}}$</td>
<td>$R^+_{\text{d}}$ degradation</td>
<td>1.7*</td>
</tr>
<tr>
<td>$k_{\text{ps1}}/k_{\text{ps2}}$</td>
<td>$R^+_{\text{ds}}$ degradation</td>
<td>1.7*</td>
</tr>
<tr>
<td>$k_{\text{ps2}}/k_{\text{ps1}}$</td>
<td>Formation of plus-strand RNA</td>
<td>1.7*</td>
</tr>
<tr>
<td>$k_{\text{ds}}$</td>
<td>$E^+_{\text{ds}}$ degradation</td>
<td>0.001–0.02‡ (molecule⁻¹)</td>
</tr>
<tr>
<td>$k_{\text{ds}}/k_{\text{ps1}}$</td>
<td>$R^+_{\text{ds}}$ formation</td>
<td>200†</td>
</tr>
</tbody>
</table>

* Obtained from the literature as explained in Materials and Methods; †, obtained from simulation as explained in Results; ‡, obtained from sensitivity analysis as explained in Results. Half-life values are indicated in parentheses.
FIG. 2. Simulation of subgenomic HCV RNA replication in Huh-7 cells. To explore the impact of each unknown parameter on subgenomic HCV kinetics, from time of transfection (t = 0) to steady state, we varied one or more chosen parameters within a given range, while all other parameters were maintained fixed as follows: $k_1 = 80 \text{ h}^{-1} \text{molecule}^{-1}$, $k_{in} = 0.2 \text{ h}^{-1}$, $k_{out} = 1.3 \times 10^{-3} \text{ h}^{-1}$, $k_2 = 0.6 \text{ h}^{-1}$, $k_2 = 100 \text{ h}^{-1}$, $k_{p} = k_{out} = 1.7 \text{ h}^{-1}$, $k_2 = 0.02 \text{ h}^{-1}$ molecule$^{-1}$, $k_s = 4 \text{ h}^{-1}$ molecule$^{-1}$, $k_{np} = 0.04 \text{ h}^{-1}$, $k_{E} = 0.04 \text{ h}^{-1}$, $k_{p} = 10 \text{ h}^{-1}$, $k_{p} = 0.07 \text{ h}^{-1}$, $k_{p} = 0.06 \text{ h}^{-1}$, $k_{p} = 0.13 \text{ h}^{-1}$, $k_5 = 0.06 \text{ h}^{-1}$, $k_{T} = 0.015 \text{ h}^{-1}$, $R_{b} = 700 \text{ ribosome complexes}$, and $R_{p}^{vir}(0) = 500 \text{ plus-strand RNA copies}$. (A) Within a range of 1 to 1,000 available ribosome complexes per cell ($R_{b}$), the model reached steady-state characteristics when $R_{b} = 500$. We found that the steady-state HCV NS5B polymerase level (green line) increases with higher $R_{b}$ numbers. The total plus- and minus-strand RNAs at steady state are shown as black and red lines, respectively. (B) We chose $k_{E}$ within the ranges $5 \times 10^{-6} \text{ to } 2 \times 10^{-4} \text{ h}^{-1}$ and $0.01 \text{ to } 0.09 \text{ h}^{-1}$, respectively. We found that the ratio $(R_{p} + R_{p}^{vir} + R_{ds})/(R_{p}^{vir} + T_{c})$, which is the ratio of total plus-strand RNAs inside and outside of the VMS (thick line), and the ratio of total plus-strand RNA to total minus-strand RNA ($R_{p}^{vir}/R_{b}^{vir}$, thin line) increase with the ratio $\frac{\mu_{ds}/R_{E}}{\mu_{ds}/k_{E}}$ between the rate of replicative intermediate degradation and the rate of polymerase transport into the VMS. To obtain a 1:1 ratio of $(R_{p} + R_{p}^{vir} + R_{ds})/(R_{p}^{vir} + T_{c})$ and a 10:1 ratio of $R_{p}^{vir}/R_{b}^{vir}$, the ratio $\mu_{ds}/R_{E}$ needs to be about $10^4$. A ratio of $-6$ of total plus-strand RNA to total minus-strand RNA in VMS is found with many different $\mu_{ds}$ and $R_{E}$ rates (dashed line). (C) We chose $k_{E}$ and $k_{E}$ within the ranges $0.004 \text{ to } 0.02 \text{ h}^{-1}$, and $0.08 \text{ to } 4.0 \text{ h}^{-1}$ and $0.004 \text{ to } 0.02 \text{ h}^{-1}$, respectively. The ratio of total plus-strand RNA to total minus-strand RNA increases with the ratio $k_{s}/k_{E}$ of formation rates of minus-strand to plus-strand replication complexes. While $k_{s}/k_{E} = 1$ does not allow for a 10:1 total plus-strand-to-minus-strand asymmetry (thick line), and 6:1 total plus-strand-to-minus-strand asymmetry in VMS (thin line), $k_{s}/k_{E} = 200$ will. (D) We checked whether the total plus-strand-to-minus-strand asymmetry can be generated with different synthesis rates of plus- and minus-strand RNA, i.e., $k_{p}/R_{b}$ > $k_{in}$. We assumed the same formation rates ($k_{s} = k_{E} = 0.02 \text{ h}^{-1}$) of plus-strand and double-strand replicative intermediate complexes ($R_{p}^{vir}$ and $R_{ds}^{vir}$, respectively). Interestingly, although 50- to 500-fold faster synthesis rates for the plus-strand RNA than for the minus-strand RNA generate larger
needed to set the ratio $k_{\text{pul}}/k_{\text{pout}}$ of the rates at which plus-strand RNA is transported into and out of the VMS to 1, set $k_{\text{pul}} = 0.2$ h$^{-1}$, and set the ratio $\mu_{\text{dil}}/k_{\text{Ein}}$ of the rates at which RIBs complexes degrade ($\mu_{\text{dil}}$) and NSSB molecules are transported into the VMS ($k_{\text{Ein}}$) at $\sim 1.0 \times 10^4$ (Fig. 2B). Varying the other unknown parameters over a large range, as given in Table 1, did not significantly affect the 1:1 ratio between total plus-strand RNA in and out of the VMS. We thus fixed $\mu_{\text{dil}}/k_{\text{Ein}} = 1.0 \times 10^4$ and $k_{\text{pul}}/k_{\text{pout}} = 0.2$ h$^{-1}$ for the rest of our analysis.

**Plus-strand HCV RNA levels at steady state.** We found that $k_{\text{Ein}}$ and $\mu_{\text{pov}}$ are the parameters that most affect the steady-state level of total ($R_{\text{pov}}^\text{Tot} = R_{\text{pov}}^\text{VMS} + R_{\text{pov}} + T_{\text{c}} + R_{\text{ip}} + R_{\text{ids}} + R_{\text{p}}$) plus-strand RNAs in a cell. Nonetheless, ranges of $\mu_{\text{pov}}$ (0.06 to 15.0 h$^{-1}$) and $k_{\text{Ein}}$ (3.8 $\times$ 10$^{-6}$ to 6.0 $\times$ 10$^{-5}$ h$^{-1}$) allow a steady state with the observed plus-strand RNA level (not shown).

**Plus-strand and minus-strand HCV RNA asymmetry levels.** The overall plus-strand RNA ($R_{\text{pov}}^\text{Tot} = R_{\text{pov}}^\text{VMS} + R_{\text{pov}} + T_{\text{c}} + R_{\text{ip}} + R_{\text{ids}} + R_{\text{p}}$) to minus-strand RNA ($R_{\text{Ids}}^\text{Tot} = R_{\text{Ids}} + R_{\text{p}} + R_{\text{ids}}$) ratio ($R_{\text{pov}}^\text{Tot}/R_{\text{Ids}}^\text{Tot}$) increases with the ratio $\mu_{\text{dil}}/k_{\text{Ein}}$ (Fig. 2B) and the ratio $k_{\text{p}}/k_{\text{d}}$ of rates at which the dsRNA and plus-strand replicative intermediate complexes are formed ($R_{\text{ids}}$ and $R_{\text{pov}}$, respectively). To obtain an approximate overall 10:1 plus-strand/minus-strand ratio (with the above-estimated parameters values), we need to assume that the formation rate of $R_{\text{p}}$ complexes is $\sim 200$-fold faster than the formation rate of $R_{\text{p}}$ (500,000), and that $k_{\text{p}}$, the rate constant for formation of $R_{\text{p}}$, is between 0.004 and 0.02 h$^{-1}$ molecule$^{-1}$ (Fig. 2C).

If we restrict this analysis to the VMS, the ratio of plus-strand RNA ($R_{\text{pov}}^\text{VMS} = R_{\text{pov}} + R_{\text{ip}} + R_{\text{ids}}$) to minus-strand RNA ($R_{\text{Ids}}^\text{Tot} = R_{\text{Ids}} + R_{\text{p}}$) also increases with the ratio $k_{\text{p}}/k_{\text{d}}$ (Fig. 2C). To obtain the approximate 6:1 plus-strand/minus-strand ratio in the VMS (50), $k_{\text{p}}/k_{\text{d}}$ should again be large: ca. 200 to 300. However, to also attain an overall ratio of 10:1, we fixed $k_{\text{p}}/k_{\text{d}}$ at 200 for the rest of our analysis.

Theoretically, the observed $R_{\text{pov}}^\text{Tot}/R_{\text{Ids}}^\text{Tot}$ ratio can be governed by significantly faster synthesis of the nascent plus-strand RNA relative to the synthesis of nascent minus-strand RNA (i.e., $k_{\text{ip}} > > > k_{\text{m}}$, assuming that the formation rates of $R_{\text{ip}}$ and $R_{\text{ids}}$ are equal, $k_{\text{d}} = k_{\text{p}}$). However, even a 500-fold difference between the plus- and minus-strand RNA synthesis rates did not generate a 10:1 $R_{\text{pov}}^\text{Tot}/R_{\text{Ids}}^\text{Tot}$ ratio (Fig. 2D). It appears that faster synthesis of the plus-strand RNA relative to the minus-strand RNA leads to more free dsRNA, $R_{\text{pov}}$, than free plus-strand RNA, $R_{\text{ip}}$, and to higher numbers of plus-strand replicative intermediate complexes than double-strand replicative intermediate complexes (Fig. 2D). The more plus-strand replicative intermediate complexes that are formed, the more minus strands that are synthesized, and this eventually leads to an $R_{\text{pov}}^\text{Tot}$/$R_{\text{Ids}}^\text{Tot}$ of $\sim 1$. Thus, faster plus-strand synthesis cannot generate the observed 10:1 ratio (Fig. 2D).

**Time to reach steady state.** After achieving a plus-strand/minus-strand ratio of 10:1 and a plus-strand RNA level of 900 to 5,000 copies/cell, we sought to understand how to adjust the remaining parameters [$k$, $k_3$, $k_5$, $\mu_{\text{pov}}$, $\mu_{\text{vcr}}$, and $R_{\text{pov}}^\text{Tot}(0)$] so that the time it takes the system to reach steady state is about 48 h. Assuming the fastest plus and minus intermediate complex formation rates consistent with the above-mentioned range, i.e., $k_3 = 0.02$ h$^{-1}$ molecule$^{-1}$ and $k_5 = 4.0$ h$^{-1}$ molecule$^{-1}$, the number of plus strands at the time of transfection, $R_{\text{pov}}^\text{Tot}(0)$, the degradation rate constant of translation complexes ($\mu_{\text{pov}}$), and the formation rate constant of translation complexes ($k_3$) affect the time it takes to attain steady state. When $R_{\text{pov}}^\text{Tot}(0)$ is $\sim 500$ copies/cell, $\mu_{\text{pov}}$, $k_3$, and $k_5$ are $< 0.02$ h$^{-1}$, and 80 h$^{-1}$ molecule$^{-1}$, both strands will reach steady state in about 48 h (Fig. 2E). In Fig. 2F we show an inverse correlation between $R_{\text{pov}}^\text{Tot}(0)$ and the time it takes to attain steady state. In addition, it is likely that the degradation rate of free NSSB in VMS ($\mu_{\text{vcr}}$) is slower than in cytoplasm ($\mu_{\text{pov}} = 0.06$ h$^{-1}$) due to the protection of the VMS from protease activity. However, $\mu_{\text{vcr}}$ within the range 0.01 to 0.006 h$^{-1}$ and the polyprotein cleavage rate $k_3$ within the range 0.2 to 0.4 h$^{-1}$ do not affect the rate of attainment of the RNA steady state (not shown).

To verify the robustness of the estimated-parameter-value ranges, 1,000 combinations of the unknown parameters were chosen randomly within the range given in Table 1. We found that ca. 70% of the 1,000 random parameter sets (Table 1, third column) led to a steady state consistent with the experimental observations criteria given in Materials and Methods (Fig. 3A). Among these 700 parameter sets, only 600 (i.e., 60% of the 1,000 random parameter sets) attain an RNA steady-state by 48 h posttransfection (data not shown). Thus, these 600 sets that fulfill the experimental observations define a new, narrower, range for each parameter (Table 1, fourth column).

We then generated 1,000 random parameter sets within the new ranges and found that more than 99% lead to a steady state, in agreement with experimental observations (Fig. 3B). For example, starting from the initial ranges for $R_{\text{ids}}^\text{VMS}$ (500 to 1,000 ribosome complexes) and $R_{\text{ids}}^\text{Tot}(0)$ (10 to 1,000 plus-strand RNA copies), we found that to be consistent with the set of experimental observation in Materials and Methods, $R_{\text{ids}}^\text{VMS}$ and $R_{\text{ids}}^\text{Tot}(0)$ should be between 650 to 910 and 340 to 830, respectively. All other new parameter ranges are shown in Table 1, column 4.

Due to the lack of experimental data on the number of plus strands at the time of transfection and the lack of detailed kinetic information about the growth rate of plus and minus strands from transfection to steady state, we cannot estimate numbers of $R_{\text{p}}$ complexes (thick red line) than of $R_{\text{ids}}$ complexes (thick black lines), the total plus-strand RNA (thin black line) is approximately equal to the total minus-strand RNA (thin red line). Free plus-strand RNA ($R_{\text{p}}$) is represented in blue. The curve for $R_{\text{p}}$ superimposes on the curve for $R_{\text{pov}}^\text{Tot}$ since almost all of the minus-strand RNA is in double-stranded complexes. (E) With 10 subgenomic plus-strand RNAs successfully transfected ($R_{\text{ids}}^\text{Tot}(0) = 10$ per cell (thin lines)), total plus-strand RNA (black line), minus-strand RNA (red line), and NSSB molecules (green line) reach steady state in about 300 h. However, with $R_{\text{pov}}^\text{Tot}(0) = 500$ (thick lines), the steady state is attained in about 50 h. The total plus-strand RNA level at steady state is not affected by different initial $R_{\text{pov}}^\text{Tot}$ numbers. (F) We tested how the plus-strand RNA copy numbers (1 to 1,000) at time of transfection, $R_{\text{pov}}^\text{Tot}(0)$, affect the time to attain steady state. Only when $R_{\text{pov}}^\text{Tot}(0)$ was $> 7$ was a steady state attained, whereas $R_{pov}^\text{Tot}(0)$ at $< 7$ led to elimination of viral RNA (not shown). Higher initial $R_{pov}^\text{Tot}$ numbers lead to faster attainment of steady-state.
FIG. 3. Sensitivity analyses for the model of subgenomic HCV RNA replication in Huh-7 cells. One thousand parameter sets within the parameter ranges given in Table 1 (column 3) were randomly chosen. (A) Within these parameter ranges we found that: (i) ~20% of parameter sets led to a total plus-strand RNA steady state of ~900 copies/cell or HCV RNA elimination, (ii) ~8% led to a total plus-strand RNA steady state of >900 copies/cell but NS5B molecules of ~8 × 10^3, (iii) ~70% (700) of parameter sets generated steady states of between 900 and 5,000 total plus-strand RNAs and ~8 × 10^4 to ~4 × 10^5 NS5B molecules, and (iv) ~2% led to a total plus-strand RNA steady state of >5,000 copies/cell (not shown). Among these 700 parameter sets only 85% (600 sets) attained an RNA steady-state in 48 h posttransfection (data not shown). (B) Using the interquartile ranges for each parameter in those 600 sets (Table 1, column 4), we found that more than 99% of 1,000 simulations using randomly chosen parameters within these ranges of the parameter led to steady states consistent with the characteristics given in Materials and Methods. Mean values of total plus-strand RNA, minus-strand RNA, and NS5B molecules are shown in black, red, and green filled circles, respectively. Vertical lines represent three standard deviations.

DISCUSSION

We have developed a kinetic model of HCV RNA replication in the absence of viral particle formation, as occurs in subgenomic replicon systems. Our results are in agreement with the recent experimental results of Quinkert et al. (50) and provide parameter ranges for the relevant kinetic parameters. Our model estimates that about 7 × 10^3 ribosomes are involved in generating millions of HCV NS5B-polymerase molecules in a Huh-7 cell and suggests that the observed 10:1 asymmetry of plus-strand to minus-strand RNA levels can be explained by a higher promoter strength or, as we call it here, a higher-affinity (200-fold) interaction of HCV NS5B-polymerase complexes with minus-strand RNA over the plus-strand RNA in order to start HCV RNA synthesis.

In our model, we assumed that plus-strand RNAs initially serve as templates to synthesize the NS5B polymerase and other essential proteins (59). We also assumed that plus-strand RNAs involved in translation cannot be templates for simultaneous RNA synthesis, as was previously shown for poliovirus replication (20). Because it is assumed that the positive-strand RNAs must be used for translation prior to RNA replication (1), HCV might have a mechanism, yet to be determined, to downregulate translation to begin RNA synthesis. However, in the model developed here translation and replication of HCV RNA coexist, although there is a competition between the two processes, with any given HCV RNA strand either being translated or replicated.

For simplicity, we assumed that 10 ribosomes bind to plus-strand RNA simultaneously and that the polysomes dissociate after each viral polyprotein is synthesized, as was previously assumed in models of Qβ replication (16, 26). We show in the parameter values precisely. However, our analysis provides plausible parameter ranges (Table 1).

**Model without VMS.** We sought to understand whether the observed RNA steady-state can be reached without the VMS. We thus simplified our model so that HCV RNA synthesis and viral polyprotein translation occur in the same compartment. Briefly, equations 1 and 5 were summed and \( k_{Pin} \), \( k_{Ein} \), \( k_{Pout} \), and \( \mu_P \) were set to zero. In addition, equations 4 and 7 were summed, and \( k_{Ein} \) and \( \mu_E \) were set to zero. To explore the impact of each remaining parameter on subgenomic HCV kinetics, from the time of transfection (t = 0) to steady state, we varied one or more chosen parameters within a given range, while all other parameters were maintained fixed as indicated in the legend to Fig. 2 (except for \( k_j \) and \( R_{ibo}^{Tot} \), which needed to be varied in order to allow RNA amplification to the observed steady-state level). Simulation results of this model within large-parameter-value ranges (e.g., 0.1 < \( \mu_{Rib} \) < 1.5 h⁻¹, 0.2 < \( k_j \) < 100 h⁻¹ molecule⁻¹, and 2 < \( R_{ibo}^{Tot} \) < 30 ribosome complexes) were in agreement with the observed RNA steady-state levels but were not consistent with the observation of millions of free NS5B molecules in cytoplasm. Typically, we found fewer than 100 free NS5B molecules. The model also predicts that only a few ribosome complexes (\( R_{ibo}^{Tot} \approx 30 \)) are needed to attain realistic RNA steady-state levels within large-parameter-value ranges (not shown). This result is reasonable; without a VMS the total plus-strand RNA in the cell can be involved in HCV polyprotein translation. To increase the predicted number of NS5B molecules, one needs to increase the number of available ribosome complexes. However, choosing a \( R_{ibo}^{Tot} \) value of greater than ~30 led to 10- to 100-fold-higher RNA steady-state levels than are observed experimentally.

For simplicity, we assumed that 10 ribosomes bind to plus-strand RNA simultaneously and that the polysomes dissociate after each viral polyprotein is synthesized, as was previously assumed in models of Qβ replication (16, 26). We show in the
Appendix that one can use this simplified model with confidence only if the rate for ribosome attachment to a free plus-strand RNA is lower than the rate constant for subsequent ribosomes to attach. This assumption may be plausible in light of RNA helicase function within a ribosome that needs first to unwind the secondary structure of the mRNA (25), which then might lead to a faster attachment of the next ribosome. In addition, to correctly account for the rate of translation with 10 ribosomes, we assumed the virus polyprotein elongation rate is 10 times faster than the estimated elongation rate per ribosome in eukaryotes (see Materials and Methods).

The total ribosome number and the fraction of available ribosomes for HCV replication in an Huh-7 cell have not yet been defined. However, it is plausible that the number of ribosomes in an Huh-7 cell is between $2 \times 10^4$, as estimated in prokaryotic organisms (E. coli [16]), and $6 \times 10^6$ ribosomes, as found in eukaryotic cells (49). According to our model, we estimated that approximately $6 \times 10^3$ ribosomes are available for HCV replication, corresponding to ca. 0.1% of the total number of ribosomes in a eukaryotic cell.

In the early 1960s, Spiegelman (54) and others questioned how a single-plus-strand virus RNA genome (e.g., Qb phage) can be amplified among thousands of host mRNA molecules present in a bacterium. It was found that the Qb replicase has a specific template affinity to the viral genome, thus allowing specific amplification of the Qb genome. It is possible that other single-stranded RNA viruses (e.g., HCV, poliovirus, and others [53]), which replicate in eukaryotic cells, developed additional mechanisms for efficient RNA amplification by co-localization of their replicative intermediates (including their replicase) in enclosed VMS. In agreement with that, we implement in our model (equations 1 to 9) the existence of VMS and generated with our model the observed subgenomic HCV steady-state characterizations. Interestingly, without the VMS, if more than ~30 ribosome complexes (i.e., 300 ribosomes) are available to support HCV replication, then significantly higher RNA steady states than we observed can occur. Thus, the VMS might restrain viral amplification and prevent host cell damage. Other possible roles for the VMS include hiding dsRNA molecules from the innate immune system of the host cell (19) and/or serving as a switch for slowing down translation in favor of HCV RNA replication.

Three major species of Flavivirinae RNAs have been described in cell-free systems (2, 4, 10, 57): dsRNA, dsDNA and recently synthesized plus- or minus-strand RNAs, and single-stranded plus- and minus-strand RNAs. However, unlike other Flaviviridae (11, 21, 60) double-stranded species (dsDNA and/or dsRNA plus recently synthesized plus- or minus-strand RNA) have not been detected during HCV replication in cell culture (24, 28, 35) or in liver biopsies (7). We predict that dsRNA species and dsDNA plus recently synthesized plus- or minus-strand RNA species are <3% and <10% of the total plus-strand RNA per cell, respectively, which may make it difficult to detect using current assays. Thus, although the exact strategy of HCV replication in Huh-7 cells is not yet firmly established, we assumed a double-stranded strategy in our model. However, our model can be simply modified to involve a single-stranded replication strategy (not shown).

The mechanism by which the asymmetry (~10:1 ratio) between plus- and minus-strand HCV RNA levels is gained in replications (35, 50) or in liver cells (9, 27) is still unknown. It might be that host factors, required for synthesizing minus-strand RNA, are responsible for this asymmetry or that there are different rate constants for the production of plus- and minus-strand RNA. It was previously shown by Eigen et al. (16) that this asymmetry can be caused simply by the fact that Qb replicase has to compete for plus-strand RNA with ribosomes and coat protein, whereas minus-strand RNAs are free for the production of plus-strand RNA. However, since HCV RNA synthesis probably occurs in VMS (15, 22), we do not implement in our model a competition between HCV replicase and ribosomes. Since host factors responsible for this asymmetry have not been identified, only different rate constants for the production of plus- and minus-strand RNAs were examined in our model. Interestingly, our model predicts that whereas different synthesis-rate constants of plus- and minus-strand RNAs do not contribute to the observed asymmetry, a higher-affinity (~200-fold) interaction of minus-strand RNA over plus-strand RNA with the NS5B polymerase-containing RNA replicase, in order to start HCV RNA synthesis, does contribute significantly. In addition, the latter higher affinity (>200-fold) leads to ~6:1 plus-strand/minus-strand ratio in VMS, in agreement with recent experimental results (50).

Our prediction for a higher-affinity interaction of the NS5B containing replicase with minus-strand RNA than with plus-strand RNA is in agreement with an observation by Reigadas et al. (52), who showed in vitro that the 3’-terminal region of minus-strand RNA was preferentially bound by purified HCV NS5B polymerase over the 3’ end of plus-strand RNA. However, it is likely the template preferences and the resulting asymmetric RNA replication will involve higher-order RNA interactions, such as those identified in the NS5B coding region (17, 29, 62), which function through a “kissing interaction” with a loop sequence in the 3’NTR. Additional viral and cellular proteins are also likely to participate in determining template preference (38; D. Quinkert et al., 13th Int. HCV Conf., abstr. 212, 2006).

Recent experimental results of Quinkert et al. (50) indicated that each HCV replication complex is composed of multiple copies of HCV NS proteins. These authors suggested that the huge excess of NS proteins is required to build up the viral replication complexes and that only <0.1% of them are required to be enzymatically active. Since in our model we only keep track of the enzymatically active NS5B molecules, our results are in agreement with their observations. In addition, the excess of NS5B molecules seen in our model also implies an excess of other NS viral proteins, since each HCV polyprotein (P) is cleaved into one copy of each HCV NS protein.

According to the model, higher numbers of plus-strand RNAs at the time of transfection lead to faster attainment of the RNA steady state but do not change its magnitude. Thus, the RNA amplification from transfection to steady state is inversely correlated with the number of plus-strand RNAs that enters the cell by transfection, in agreement with observations of Lohmann et al. (34). However, small numbers of transfected RNAs (see the legend to Fig. 2F) may lead to the elimination of HCV RNA. Of note, in the model we do not consider that some of the transfected RNAs might be defective. If this were the case then if a few RNAs were transfected it would be possible that no viral RNA amplification occurred.
In summary, we have developed a mathematical model for subgenomic HCV replication within HuH-7 cell that uses a double-stranded strategy for RNA amplification. Our model suggests a mechanism by which the ratio of plus-strand to minus-strand RNA is regulated and shows that RNA replication occurring in a membrane compartment has advantages for the HCV life cycle. Now that fully permissive HCV replication systems have been developed (31, 58, 63), the next step will be to incorporate virus production and infection to create one comprehensive model of the complete HCV life cycle. The model developed here can serve as an important tool in understanding HCV replication mechanisms and may prove useful in designing and evaluating new antivirals for use against HCV.

**APPENDIX**

**Modeling multiple ribosomes attaching to HCV mRNA.** To explore the sequential attachment of 10 ribosomes to each HCV mRNA (59), we developed the following model:

\[
R_p^{co} + R_{ibo} \rightarrow R_1
\]

Let \( R_p \) represents the number of polysomes consisting of \( i \) ribosomes attached to an HCV mRNA (plus-strand RNA). Free plus-strand RNA molecules, \( R_p^{co} \), are converted at rate \( k_1 \) into \( R_1 \) due to ribosome attachment. Then \( R_1 = 1, \ldots, 8 \) disappear at rate \( k_1^* \) by additional ribosome attachment, forming \( R_{1+i} \). Finally, \( R_9 \) disappears at rate \( k_2 \) by forming the translation complex, \( T_e \), and reappears at rate \( k_2^* \) when the translation is complete and the newly synthesized polyprotein, \( P \), and its related ribosome dissociate from the HCV mRNA.

We further assume that once plus-strand RNA attaches to a ribosome and/or becomes a polysome, it will remain in this complexed state until its degradation, with an average rate of \( \mu_2 \). Under these assumptions this model can be converted to the following differential equations:

\[
\frac{dR_1}{dt} = k_1 R_{ibo} R_p^{co} - k_1^* R_{ibo} R_1 - \mu_2 R_1
\]

\[
\frac{dR_{i+1}}{dt} = k_1^* R_{ibo} R_i - k_1^* R_{ibo} R_{i+1} - \mu_2 R_{i+1}, \quad (i = 1, \ldots, 7)
\]

\[
\frac{dR_9}{dt} = k_2^* R_{ibo} R_8 + k_2 T_e - k_2^* R_{ibo} R_9 - \mu_2 R_9
\]

These equations can be easily included in our simplified model, equations 1 to 9, necessitating modifying equations 1 and 2 as follows:

\[
\frac{dR_p^{co}}{dt} = k_{Pin} R_p - k_1 R_{ibo} R_p^{co} - k_{Pin} R_p^{co} - \mu_p R_p^{co}
\]

\[
\frac{dT_e}{dt} = k_4^* R_{ibo} R_9 - k_2 T_e - \mu_2 T_e
\]

In addition, we modified the equation in Materials and Methods for the free ribosomes (\( R_{ibo} \)) as follows:

\[
R_{ibo} = R_{ibo}^{Tot} - 10 T_e - \sum_{i=1}^{9} R_i
\]

We found that if one assumes that the rate constant \( k_1^* \), for the first ribosome to attach is lower than the rate constant \( k_1^\*)*, for subsequent ribosomes to attach, then the simplified model and complex model give similar results (not shown). However, when \( k_1^* \geq k_1^\*)* \) the model results, within the parameter ranges given in Table 1, are not in agreement with the experimental data (not shown).

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**REFERENCES**


